Tutorial 6
Advanced Graph Theory

August 27, 2013

1. Prove or disprove: Every tree has atmost one perfect matching.
2. Prove that every maximal matching in a graph G has size at least half the size of a maximum matching.
3. Use the Konig-Egervary Theorem to prove that every bipartite graph G has a matching of size at least $e(G)/\Delta(G)$. Use this to conclude that every subgraph of $K_{n,n}$ with more than $(k-1)n$ edges has a matching of size at least k.
4. Determine the maximum number of edges in a simple bipartite graph that contains no matching with k edges and no star with l edges. (Isaak)

5. Consider a bipartite graph G with partite sets X and Y. For any subset of vertices S, let $N(S)$ denote the set of vertices such that there is an edge from vertex in S to some vertex in $N(S)$. Show that if $|N(S)| \geq |S|d$ for every subset S of X and some fixed positive integer d, then G has a matching of size, $|X| - d$.

6. Let G be a simple graph in which the sum of the degrees of any k vertices is less than $(n - k)$. Prove that every maximal independent set in G has more than k vertices.